

Code Generation for Data Processing

Lecture 5: Analyses and Transformations

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Winter 2025/26

Program Transformation: Motivation

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- ▶ Generating “better” IR code on first attempt is expensive
 - ▶ What parts are actually used? How to find out?
- ▶ Transformation to “better” code must be done *somewhere*

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 - ▶ Many transformations are driven by heuristics

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- ▶ Allows for more pragmatic/simple code
- ▶ Generating “better” IR code on first attempt is expensive
 - ▶ What parts are actually used? How to find out?
- ▶ Transformation to “better” code must be done *somewhere*
- ▶ Optimization is a misnomer: we don’t know whether it improves code!
 - ▶ Many transformations are driven by heuristics
- ▶ Many types of optimizations are well-known¹⁶

¹⁶FE Allen and J Cocke. *A catalogue of optimizing transformations*. 1971. .

Dead Block Elimination

- ▶ CFG not necessarily connected
- ▶ E.g., consequence of optimization
 - ▶ Conditional branch $\rightarrow$ unconditional branch

Dead Block Elimination

- ▶ CFG not necessarily connected
- ▶ E.g., consequence of optimization
 - ▶ Conditional branch $\rightarrow$ unconditional branch
- ▶ Removing dead blocks is trivial
 1. DFS traversal of CFG from entry, mark visited blocks
 2. Remove unmarked blocks

Optimization Example 1

```
define i32 @fac(i32 %0) {  
    br label %for.header  
for.header: ; preds = %for.body, %1  
    %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
    %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
    %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
    %cond = icmp sle i32 %i, %0  
    br i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
    %a.new = mul i32 %a, %i  
    %b.new = add i32 %b, %i  
    %i.new = add i32 %i, 1  
    br label %for.header  
exit: ; preds = %for.header  
    %absum = add i32 %a, %b  
    ret i32 %a  
}
```

Simple Dead Code Elimination (DCE)

- ▶ Look for trivially dead instructions
 - ▶ No users or side-effects
 - ▶ Calls *might* be removed
- 1. Add all instructions to work queue
- 2. While work queue not empty:
 - 2.1 Check for deadness (zero users, no side-effects)
 - 2.2 If dead, remove and add all operands to work queue

Warning: Don't implement it this naively, this is inefficient

Applying Simple DCE

```
define i32 @fac(i32 %0) {  
  eff.: cf    br label %for.header  
  for.header: ; preds = %for.body, %1  
  users: 3    %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
  users: 2    %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
  users: 4    %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
  users: 1    %cond = icmp sle i32 %i, %0  
  eff.: cf    br i1 %cond, label %for.body, label %exit  
  for.body:   ; preds = %for.header  
  users: 1    %a.new = mul i32 %a, %i  
  users: 1    %b.new = add i32 %b, %i  
  users: 1    %i.new = add i32 %i, 1  
  eff.: cf    br label %for.header  
  exit:       ; preds = %for.header  
  users: 0    %absum = add i32 %a, %b  
  eff.: cf    ret i32 %a  
}
```

Applying Simple DCE

```
define i32 @fac(i32 %0) {  
  eff.: cf    br label %for.header  
  for.header: ; preds = %for.body, %1  
  users: 2    %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
  users: 1    %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
  users: 4    %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
  users: 1    %cond = icmp sle i32 %i, %0  
  eff.: cf    br i1 %cond, label %for.body, label %exit  
  for.body:   ; preds = %for.header  
  users: 1    %a.new = mul i32 %a, %i  
  users: 1    %b.new = add i32 %b, %i  
  users: 1    %i.new = add i32 %i, 1  
  eff.: cf    br label %for.header  
  exit:       ; preds = %for.header  
  
  eff.: cf    ret i32 %a  
}
```

Dead Code Elimination

- ▶ Problem: unused value cycles

Dead Code Elimination

- ▶ Problem: unused value cycles
 - ▶ Idea: find “value sinks” and mark all needed values as live
 - ▶ Sink: instruction with side effects (e.g., store, control flow)
 - ▶ Alternative formulation: assume instruction is dead unless proven otherwise
-
1. Only mark instrs. with side effects as live
 2. Populate work list with newly added live instrs.
 3. While work list not empty:
 - 3.1 Mark dead operand instructions as live and add to work list
 4. Remove instructions not marked as live

Applying Liveness-based DCE

```
define i32 @fac(i32 %0) {  
  br1 label %for.header  
for.header: ; preds = %for.body, %1  
  %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
  %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
  %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
  %cond = icmp sle i32 %i, %0  
  br2 i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
  %a.new = mul i32 %a, %i  
  %b.new = add i32 %b, %i  
  %i.new = add i32 %i, 1  
  br3 label %for.header  
exit: ; preds = %for.header  
  %absum = add i32 %a, %b  
  ret i32 %a  
}
```

Work list (stack)

Applying Liveness-based DCE

```
define i32 @fac(i32 %0) {  
live  br1 label %for.header  
for.header: ; preds = %for.body, %1  
    %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
    %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
    %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
    %cond = icmp sle i32 %i, %0  
live  br2 i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
    %a.new = mul i32 %a, %i  
    %b.new = add i32 %b, %i  
    %i.new = add i32 %i, 1  
live  br3 label %for.header  
exit: ; preds = %for.header  
    %absum = add i32 %a, %b  
live  ret i32 %a  
}
```

Work list (stack)

br₁
br₂
br₃
ret

Applying Liveness-based DCE

```
define i32 @fac(i32 %0) {  
live  br1 label %for.header  
for.header: ; preds = %for.body, %1  
live  %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
      %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
      %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
      %cond = icmp sle i32 %i, %0  
live  br2 i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
      %a.new = mul i32 %a, %i  
      %b.new = add i32 %b, %i  
      %i.new = add i32 %i, 1  
live  br3 label %for.header  
exit: ; preds = %for.header  
      %absum = add i32 %a, %b  
live  ret i32 %a  
}
```

Work list (stack)

br₁
br₂
br₃
%a

Applying Liveness-based DCE

```
define i32 @fac(i32 %0) {  
live  br1 label %for.header  
for.header: ; preds = %for.body, %1  
live  %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
      %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
      %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
      %cond = icmp sle i32 %i, %0  
live  br2 i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
live  %a.new = mul i32 %a, %i  
      %b.new = add i32 %b, %i  
      %i.new = add i32 %i, 1  
live  br3 label %for.header  
exit: ; preds = %for.header  
      %absum = add i32 %a, %b  
live  ret i32 %a  
}
```

Work list (stack)

br₁
br₂
br₃
%a.new

Applying Liveness-based DCE

```
define i32 @fac(i32 %0) {  
live  br1 label %for.header  
for.header: ; preds = %for.body, %1  
live  %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
      %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
live  %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
      %cond = icmp sle i32 %i, %0  
live  br2 i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
live  %a.new = mul i32 %a, %i  
      %b.new = add i32 %b, %i  
      %i.new = add i32 %i, 1  
live  br3 label %for.header  
exit: ; preds = %for.header  
      %absum = add i32 %a, %b  
live  ret i32 %a  
}
```

Work list (stack)

br₁
br₂
br₃
%i

Applying Liveness-based DCE

```
define i32 @fac(i32 %0) {  
live   br1 label %for.header  
for.header: ; preds = %for.body, %1  
live   %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
       %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
live   %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
       %cond = icmp sle i32 %i, %0  
live   br2 i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
live   %a.new = mul i32 %a, %i  
       %b.new = add i32 %b, %i  
live   %i.new = add i32 %i, 1  
live   br3 label %for.header  
exit: ; preds = %for.header  
       %absum = add i32 %a, %b  
live   ret i32 %a  
}
```

Work list (stack)

br₁
br₂
br₃
%i.new

Applying Liveness-based DCE

```
define i32 @fac(i32 %0) {  
live  br1 label %for.header  
for.header: ; preds = %for.body, %1  
live  %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
      %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
live  %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
      %cond = icmp sle i32 %i, %0  
live  br2 i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
live  %a.new = mul i32 %a, %i  
      %b.new = add i32 %b, %i  
live  %i.new = add i32 %i, 1  
live  br3 label %for.header  
exit: ; preds = %for.header  
      %absum = add i32 %a, %b  
live  ret i32 %a  
}
```

Work list (stack)

br₁

br₂

br₃

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```
define i32 @fac(i32 %0) {  
live   br1 label %for.header  
for.header: ; preds = %for.body, %1  
live   %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
        %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
live   %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
        %cond = icmp sle i32 %i, %0  
live   br2 i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
live   %a.new = mul i32 %a, %i  
        %b.new = add i32 %b, %i  
live   %i.new = add i32 %i, 1  
live   br3 label %for.header  
exit: ; preds = %for.header  
        %absum = add i32 %a, %b  
live   ret i32 %a  
}
```

Work list (stack)

br₁

br₂

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```
define i32 @fac(i32 %0) {  
live  br1 label %for.header  
for.header: ; preds = %for.body, %1  
live  %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
      %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
live  %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
live  %cond = icmp sle i32 %i, %0  
live  br2 i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
live  %a.new = mul i32 %a, %i  
      %b.new = add i32 %b, %i  
live  %i.new = add i32 %i, 1  
live  br3 label %for.header  
exit: ; preds = %for.header  
      %absum = add i32 %a, %b  
live  ret i32 %a  
}
```

Work list (stack)

br₁
%cond

Applying Liveness-based DCE

```
define i32 @fac(i32 %0) {  
live   br1 label %for.header  
for.header: ; preds = %for.body, %1  
live   %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
        %b = phi i32 [ 0, %1 ], [ %b.new, %for.body ]  
live   %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
live   %cond = icmp sle i32 %i, %0  
live   br2 i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
live   %a.new = mul i32 %a, %i  
        %b.new = add i32 %b, %i  
live   %i.new = add i32 %i, 1  
live   br3 label %for.header  
exit: ; preds = %for.header  
        %absum = add i32 %a, %b  
live   ret i32 %a  
}
```

Work list (stack)
br₁

Applying Liveness-based DCE

Work list (stack)

```
define i32 @fac(i32 %0) {  
live   br1 label %for.header  
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live   %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
live   %cond = icmp sle i32 %i, %0  
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}
```

Applying Liveness-based DCE

Work list (stack)

```
define i32 @fac(i32 %0) {  
live   br1 label %for.header  
for.header: ; preds = %for.body, %1  
live   %a = phi i32 [ 1, %1 ], [ %a.new, %for.body ]  
  
live   %i = phi i32 [ 0, %1 ], [ %i.new, %for.body ]  
live   %cond = icmp sle i32 %i, %0  
live   br2 i1 %cond, label %for.body, label %exit  
for.body: ; preds = %for.header  
live   %a.new = mul i32 %a, %i  
  
live   %i.new = add i32 %i, 1  
live   br3 label %for.header  
exit: ; preds = %for.header  
  
live   ret i32 %a  
}
```

Optimization Example 2

```
define i32 @trivial() {  
  entry:  
    br label %for.cond  
for.cond:  
  %x = phi i32 [ 0, %entry ], [ %inc, %for.inc ]  
  %cmp = icmp ugt i32 %x, 0  
  br i1 %cmp, label %for.inc, label %ret  
for.inc:  
  %inc = add i32 %x, 1  
  br label %for.cond  
ret:  
  ret i32 %x  
}
```

Sparse Conditional Constant Propagation¹⁷

- ▶ Constant folding and dead code elimination separately insufficient
- ▶ Idea: be optimistic about reachability and constantness
 - ▶ Assume block is dead/value is constant unless proven otherwise
 - ▶ Ignore edges that are not proven as executable

¹⁷MN Wegman and FK Zadeck. "Constant propagation with conditional branches". In: *TOPLAS* 13.2 (1991), pp. 181–210. .

Sparse Conditional Constant Propagation¹⁷

- ▶ Constant folding and dead code elimination separately insufficient
- ▶ Idea: be optimistic about reachability and constantness
 - ▶ Assume block is dead/value is constant unless proven otherwise
 - ▶ Ignore edges that are not proven as executable
- ▶ For every value: store lattice element
 - ▶ Lattice: $\top$ (any value) $>$ *constant* (single constant value) $>$ $\perp$ (dynamic)
 - ▶ $\top \wedge x = x$, $\perp \wedge x = \perp$, $x \wedge y = (x \text{ if } x = y \text{ else } \perp)$
- ▶ For every edge: store executability
 - ▶ ϕ -nodes: edges that are not (yet) marked executable get $\top$

¹⁷MN Wegman and FK Zadeck. "Constant propagation with conditional branches". In: *TOPLAS* 13.2 (1991), pp. 181–210. .

SCCP Algorithm

State:

- ▶ EdgeWorklist
 - ▶ InstrWorklist
 - ▶ EdgeExecutable
 - ▶ Initialize false
 - ▶ InstrLattice
 - ▶ Initialize $\top$
- ▶ EdgeWorklist $\leftarrow$ edge to entry block
 - ▶ Get item from any work list; stop when empty
 - ▶ Edge – if not yet visited, mark visited and:
 - ▶ Visited block first time: *visit* all instrs (incl. ϕ)
 - ▶ Otherwise: *visit* just ϕ -nodes
 - ▶ Instr – if block is reachable: *visit*
 - ▶ At end: replace instrs with const lattice values

SCCP Algorithm II

visit

- ▶ ϕ -node: update lattice, meet of visited edge values
- ▶ Other instructions: try constant fold
- ▶ If lattice value changed:
 - ▶ Add all users to instruction worklist
 - ▶ Branch with constant/no condition: add selected edge to worklist
 - ▶ Branch with $\perp$: add all edges to worklist

```

define i32 @fn() {
entry:
    br label %loop
loop:
    %j2 = phi i32 [ 1, %entry ], [ %j4, %ifmerge ]
    %k2 = phi i32 [ 0, %entry ], [ %k4, %ifmerge ]
    %kcond = icmp slt i32 %k2, 100
    br i1 %kcond, label %loopbody, label %ret
loopbody:
    %jcond = icmp slt i32 %j2, 20
    br i1 %jcond, label %then, label %else
then:
    %k3 = add i32 %k2, 1
    br label %ifmerge
else:
    %k5 = add i32 %k2, 1
    br label %ifmerge
ifmerge:
    %j4 = phi i32 [ 1, %then ], [ %k2, %else ]
    %k4 = phi i32 [ %k3, %then ], [ %k5, %else ]
    br label %loop
ret:
    ret i32 %j2
}

```

Apply SCCP algorithm
and fold constants.
Then, remove dead
instructions and blocks.

SCCP: Misc

- ▶ Strictly more powerful than separate constant folding + DCE
- ▶ Runtime:

SCCP: Misc

- ▶ Strictly more powerful than separate constant folding + DCE
- ▶ Runtime: $\mathcal{O}(\#CFG\text{-edges} + \#instr\text{-operands})$
 - ▶ Every CFG edge is processed at most once
 - ▶ Every def-use relation is processed at most twice
lattice can lower at most twice ($\top \rightarrow const, const \rightarrow \perp$)
- ▶ Can also be used as inter-procedural optimization
reaching into module-internal functions

Optimization Example 3

```
define i32 @foo(i32 %0, ptr %1, ptr %2) {  
    %4 = zext i32 %0 to i64  
    %5 = getelementptr i32, ptr %1, i64 %4  
    %6 = load i32, ptr %5, align 4  
    %7 = zext i32 %0 to i64  
    %8 = getelementptr i32, ptr %2, i64 %7  
    %9 = load i32, ptr %8, align 4  
    %10 = add nsw i32 %6, %9  
    ret i32 %10  
}
```

Common Subexpression Elimination (CSE) – Attempt 1

- ▶ Idea: find/eliminate redundant computation of same value

Common Subexpression Elimination (CSE) – Attempt 1

- ▶ Idea: find/eliminate redundant computation of same value
- ▶ Keep track of previously seen values in hash map
- ▶ Iterate over all instructions
 - ▶ If found in map, remove and replace references
 - ▶ Otherwise add to map
- ▶ Easy, right?

CSE Attempt 1 – Example 1

```
define i32 @foo(i32 %0, ptr %1, ptr %2) {  
  %4 = zext i32 %0 to i64  
  %5 = getelementptr i32, ptr %1, i64 %4  
  %6 = load i32, ptr %5, align 4  
  %7 = zext i32 %0 to i64  
  %8 = getelementptr i32, ptr %2, i64 %7  
  %9 = load i32, ptr %8, align 4  
  %10 = add nsw i32 %6, %9  
  ret i32 %10  
}
```

CSE Attempt 1 – Example 1

```
→ ht  define i32 @foo(i32 %0, ptr %1, ptr %2) {  
        %4 = zext i32 %0 to i64  
        %5 = getelementptr i32, ptr %1, i64 %4  
        %6 = load i32, ptr %5, align 4  
        %7 = zext i32 %0 to i64  
        %8 = getelementptr i32, ptr %2, i64 %7  
        %9 = load i32, ptr %8, align 4  
        %10 = add nsw i32 %6, %9  
        ret i32 %10  
    }
```

CSE Attempt 1 – Example 1

```
define i32 @foo(i32 %0, ptr %1, ptr %2) {  
→ ht    %4 = zext i32 %0 to i64  
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        %6 = load i32, ptr %5, align 4  
        %7 = zext i32 %0 to i64  
        %8 = getelementptr i32, ptr %2, i64 %7  
        %9 = load i32, ptr %8, align 4  
        %10 = add nsw i32 %6, %9  
        ret i32 %10  
}
```

CSE Attempt 1 – Example 1

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define i32 @foo(i32 %0, ptr %1, ptr %2) {  
→ ht    %4 = zext i32 %0 to i64  
→ ht    %5 = getelementptr i32, ptr %1, i64 %4  
→ ht    %6 = load i32, ptr %5, align 4  
        %7 = zext i32 %0 to i64  
        %8 = getelementptr i32, ptr %2, i64 %7  
        %9 = load i32, ptr %8, align 4  
        %10 = add nsw i32 %6, %9  
        ret i32 %10  
}
```

CSE Attempt 1 – Example 1

```
define i32 @foo(i32 %0, ptr %1, ptr %2) {  
→ ht    %4 = zext i32 %0 to i64  
→ ht    %5 = getelementptr i32, ptr %1, i64 %4  
→ ht    %6 = load i32, ptr %5, align 4  
dup %4  %7 = zext i32 %0 to i64  
        %8 = getelementptr i32, ptr %2, i64 %7  
        %9 = load i32, ptr %8, align 4  
        %10 = add nsw i32 %6, %9  
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- Obsolete instr. can be killed immediately, or in a later DCE

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  if.then: ; preds = %entry  
    %add1 = add i32 %a, %b  
    br label %if.end  
  if.end: ; preds = %if.then, %entry  
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→ ht      %res = add i32 %condvar, %add1  
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}
```

Instruction does not dominate all uses!

error: input module is broken!

World Domination

Domination

- ▶ Remember: CFG $G = (N, E, s)$ with digraph (N, E) and entry $s \in N$
- ▶ Dominate: $d \text{ dom } n$ iff every path from s to n contains d
 - ▶ Dominators of n : $DOM(n) = \{d \mid d \text{ dom } n\}$
- ▶ Strictly dominate: $d \text{ sdom } n \Leftrightarrow d \text{ dom } n \wedge d \neq n$
- ▶ Immediate dominator:
 $\text{idom}(n) = d : d \text{ sdom } n \wedge \nexists d'. d \text{ sdom } d' \wedge d' \text{ sdom } n$

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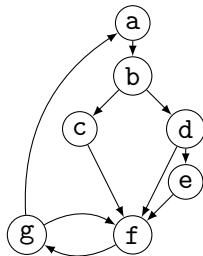
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Dominator Tree

- ▶ Tree of immediate dominators
- ▶ Allows to iterate over blocks in pre-order/post-order
- ▶ Answer $a \text{ sdom } b$ quickly

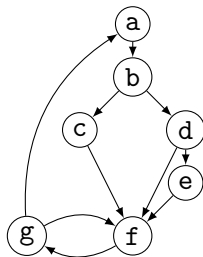
Control Flow Graph Dominator Tree



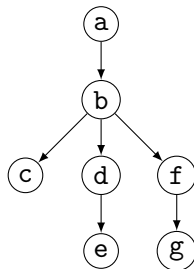
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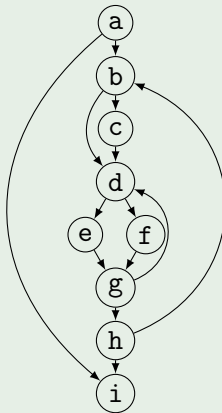
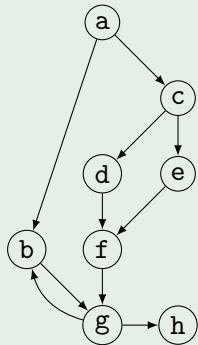


Dominator Tree



Dominator Tree – Example

Construct the dominator tree for the following CFGs (entry at a):



Dominator Tree: Construction

- ▶ Naive: inefficient (but reasonably simple)¹⁸
 - ▶ For each block: find a path from the root – superset of dominators
 - ▶ Remove last block on path and check for alternative path
 - ▶ If no alternative path exists, last block is idom

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- ▶ Lengauer–Tarjan: more efficient methods¹⁹
 - ▶ Simple method in $\mathcal{O}(m \log n)$; sophisticated method in $\mathcal{O}(m \cdot \alpha(m, n))$
($\alpha(m, n)$ is the inverse Ackermann function, grows *extremely* slowly)
 - ▶ Used in some compilers²⁰
- ▶ Semi-NCA: $\mathcal{O}(n^2)$, but lower constant factors²¹

¹⁸ES Lowry and CW Medlock. “Object code optimization”. In: *CACM* 12.1 (1969), pp. 13–22. .

¹⁹T Lengauer and RE Tarjan. “A fast algorithm for finding dominators in a flowgraph”. In: *TOPLAS* 1.1 (1979), pp. 121–141. .

²⁰Example: <https://github.com/WebKit/WebKit/blob/aabfacb/Source/WTF/wtf/Dominators.h>

²¹L Georgiadis. “Linear-Time Algorithms for Dominators and Related Problems”. PhD thesis. Princeton University, Nov. 2005. .

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 - ▶ $a.preNum < b.preNum \wedge a.postNum > b.postNum$
 - ▶ After updates, numbers might be invalid: recompute or walk tree
- ▶ Problem: dominance of unreachable blocks ill-defined $\rightsquigarrow$ special handling

CSE Attempt 2

- ▶ Option 1:
 - ▶ For identical instructions, store all
 - ▶ Add dominance check before replacing
 - ▶ Visit nodes in reverse post-order (i.e., topological order)
- ▶ Option 2 (Global Value Numbering):²⁴
 - ▶ Do a DFS over dominator tree
 - ▶ Use scoped hashmap to track available values

Does this work?

²⁴P Briggs, KD Cooper, and LT Simpson. *Value numbering*. Tech. rep. CRPC-TR94517-S. Rice University, 1997. .

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CSE: Hashing an Instruction (and Beyond)

- ▶ Needs hash function *and* “relaxed” equality
- ▶ Idea: combine opcode and operands/constants into hash value
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 - ▶ E.g. assign number to values (hence “value numbering”)
 - ▶ Alternatively: use pointer or index for instruction result operands
- ▶ Canonicalize commutative operations
 - ▶ Order operands deterministically, e.g., by number/address
- ▶ Identities: $a+(b+c)$ vs. $(a+b)+c$

More Complex Global Value Numbering

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- ▶ Hash-based approach only catches trivially removable duplicates
- ▶ Alternative: partition values into *congruence classes*
 - ▶ Congruent values are guaranteed to always have the same value
- ▶ Optimistic approach: values are congruent unless proven otherwise
- ▶ Pessimistic approach: values are not congruent unless proven
- ▶ Combinable with: reassociation, DCE, constant folding
- ▶ Rather complex, but can be highly beneficial²⁵

²⁵K Gargi. "A sparse algorithm for predicated global value numbering". In: *PLDI. 2002*, pp. 45–56.

Inlining

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- ▶ Inlining is crucial for performance in many programming languages
 - ▶ “Zero-cost” abstractions typically rely on small functions
 - ▶ Critical in e.g. C++/Rust; not too important for C

Inlining: Decision

- ▶ First: determine whether function is inlinable
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- ▶ Problem 1: ordering
 - ▶ Inlining big `a()` into `b()` might make `b()` too big to inline but inlining `b()` into `c()` might provide more opportunities
- ▶ Problem 2: cost model and thresholds
 - ▶ Estimate cost of inlining, e.g. additional code size, more branches
 - ▶ Estimate optimization gains and hotness
 - ▶ Some attempts to use machine learning here

Inlining Transformation

- ▶ Copy original function in place of the call
 - ▶ Split basic block containing function call
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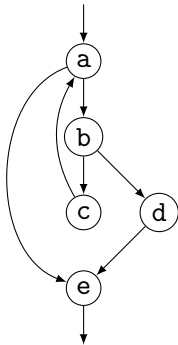
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- ▶ Move static alloca to beginning
- ▶ Dynamic alloca: save/restore stack pointer
 - ▶ Prevent unbounded stack growth in loops
 - ▶ LLVM provides `stacksave/stackrestore` intrinsics
- ▶ Exceptions may need special treatment

What is a Loop?

```
void func() {  
    while (a()) {  
        if (b()) {  
            d();  
            break;  
        }  
        c();  
    }  
    e();  
}
```

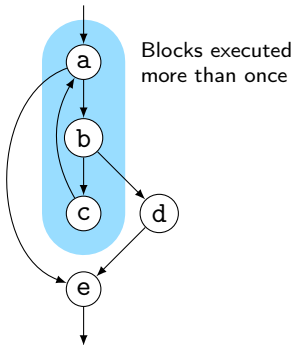
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}
```



- ▶ Loops in source code $\neq$ loops in CFG
- ▶ d is *not* part of loop: executed at most once
- ~> Need algorithm to find loops in CFG

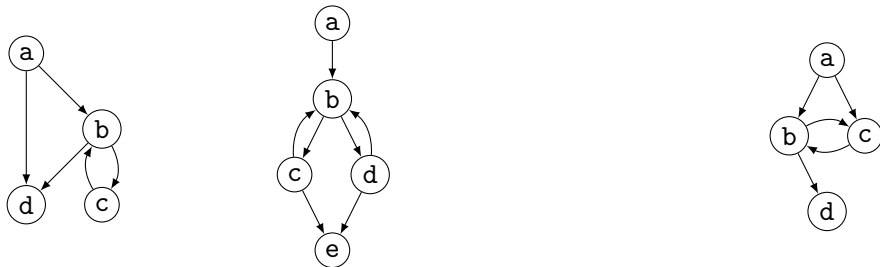
Loops

- ▶ Loop: maximal SCC L with at least one internal edge²⁷
(strongly connected component (SCC): all blocks reachable from each other)
 - ▶ Entry: block with an edge from outside of L
 - ▶ Header h : first entry found (might be ambiguous)
- ▶ Loop nested in L : loop in subgraph $L \setminus \{h\}$

²⁷P Havlak. “Nesting of reducible and irreducible loops”. In: *TOPLAS* 19.4 (1997), pp. 557–567. .

Loops

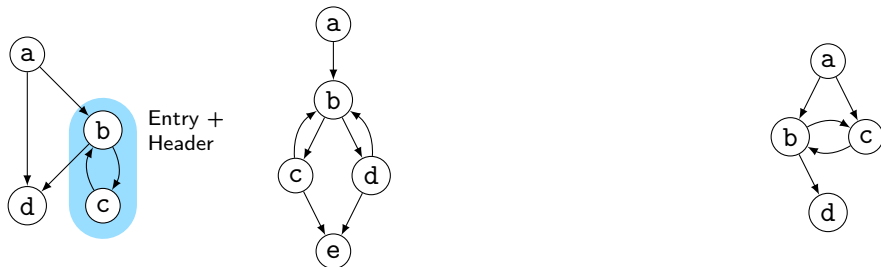
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Loops

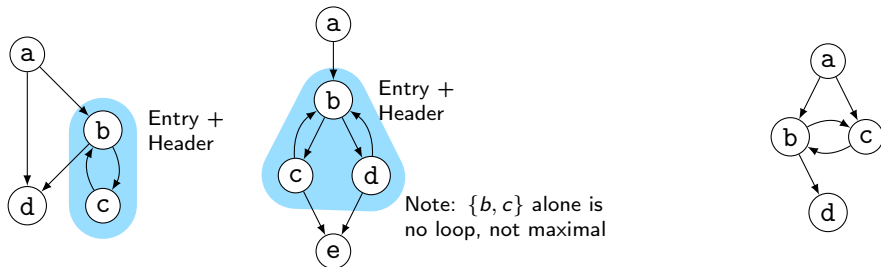
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Loops

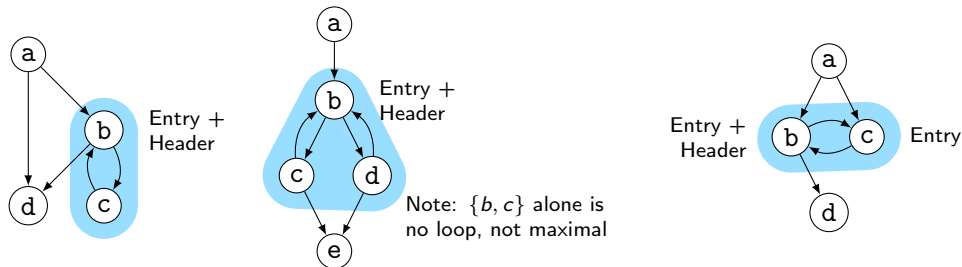
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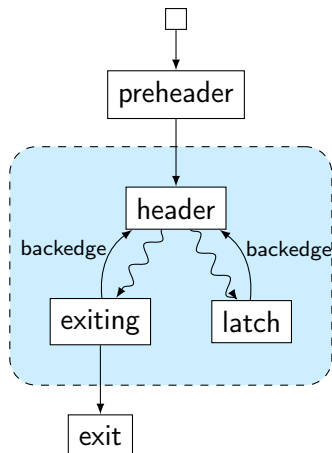
Natural Loops

- ▶ Natural Loop: loop with single entry
 - ⇒ Header is unique
 - ⇒ Header dominates all block
 - ⇒ Loop is reducible
- ▶ Backedge: edge from block to header
- ▶ Predecessor: block with edge into loop
- ▶ Preheader: unique predecessor

Formal Definition

Loop L is reducible iff $\exists h \in L . \forall n \in L . h \text{ dom } n$

CFG is reducible iff all loops are reducible



Finding Natural Loops

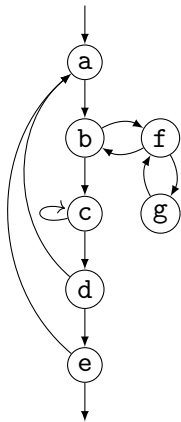
- ▶ Modified version²⁸ of Tarjan's algorithm²⁹
- ▶ Iterate over dominator tree in post order
- ▶ Each block: find predecessors dominated by the block
 - ▶ None $\rightsquigarrow$ no loop header, continue
 - ▶ Any $\rightsquigarrow$ loop header, these edges *must* be backedges
- ▶ Walk through predecessors until reaching header again
 - ▶ All blocks on the way must be part of the loop body
 - ▶ Might encounter nested loops, update loop parent

²⁸G Ramalingam. "Identifying loops in almost linear time". In: *TOPLAS* 21.2 (1999), pp. 175–188. .

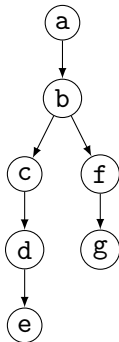
²⁹R Tarjan. "Testing flow graph reducibility". In: *STOC*. 1973, pp. 96–107. .

Finding Natural Loops: Example

Control Flow Graph



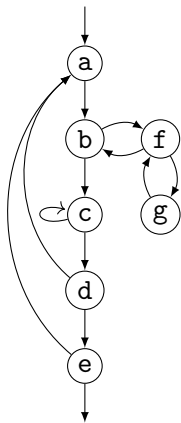
Dominator Tree



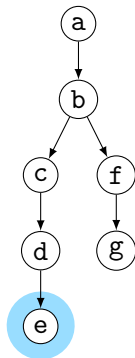
Loop Info

Finding Natural Loops: Example

Control Flow Graph



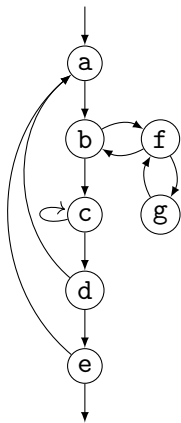
Dominator Tree



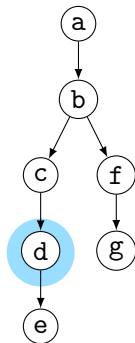
Loop Info

Finding Natural Loops: Example

Control Flow Graph



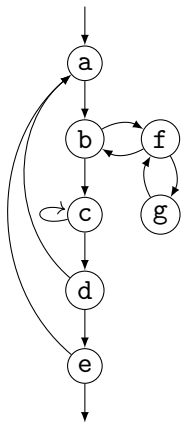
Dominator Tree



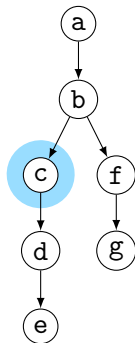
Loop Info

Finding Natural Loops: Example

Control Flow Graph



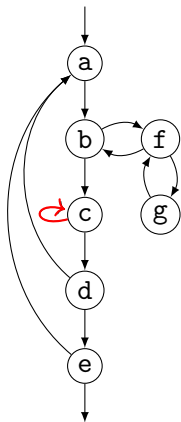
Dominator Tree



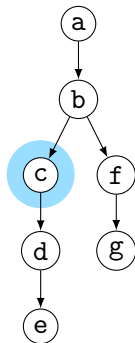
Loop Info

Finding Natural Loops: Example

Control Flow Graph



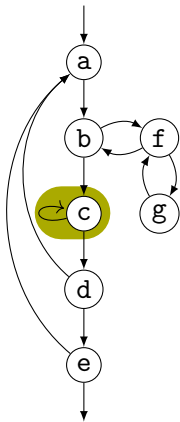
Dominator Tree



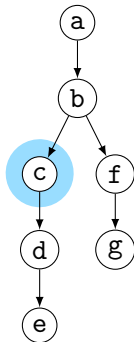
Loop Info

Finding Natural Loops: Example

Control Flow Graph



Dominator Tree

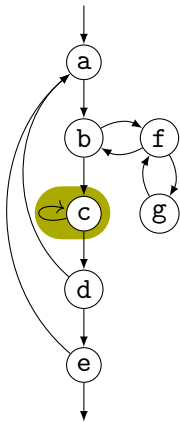


Loop Info

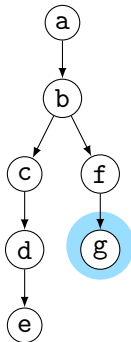
Loop **A**: {c}
header: c; parent: NULL

Finding Natural Loops: Example

Control Flow Graph



Dominator Tree

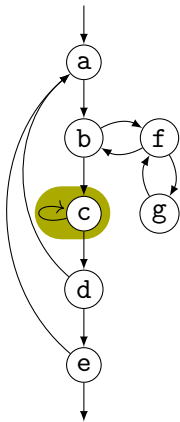


Loop Info

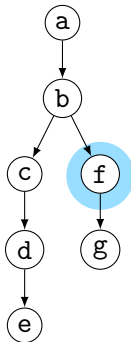
Loop **A**: {c}
header: c; parent: NULL

Finding Natural Loops: Example

Control Flow Graph



Dominator Tree

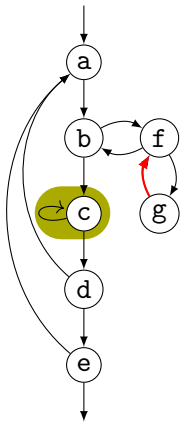


Loop Info

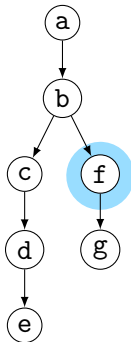
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Finding Natural Loops: Example

Control Flow Graph



Dominator Tree

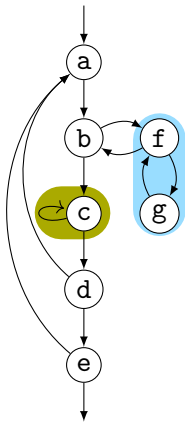


Loop Info

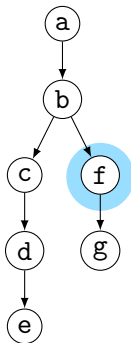
Loop **A**: {c}
header: c; parent: NULL

Finding Natural Loops: Example

Control Flow Graph



Dominator Tree



Loop Info

Loop **A**: {c}

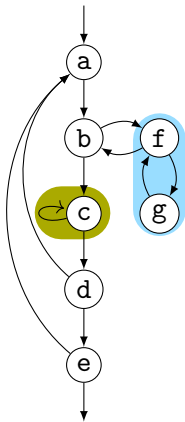
header: c; parent: NULL

Loop **B**: {f,g}

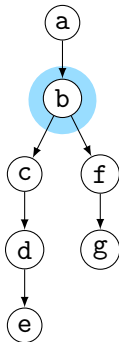
header: f; parent: NULL

Finding Natural Loops: Example

Control Flow Graph



Dominator Tree



Loop Info

Loop **A**: {c}

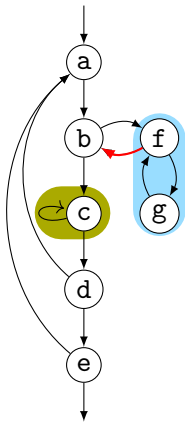
header: c; parent: NULL

Loop **B**: {f,g}

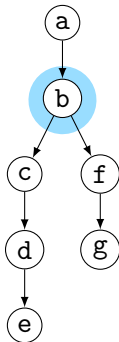
header: f; parent: NULL

Finding Natural Loops: Example

Control Flow Graph



Dominator Tree



Loop Info

Loop **A**: {c}

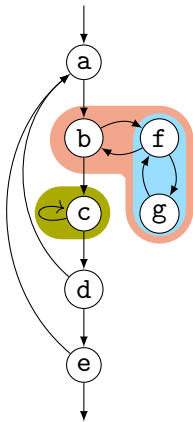
header: c; parent: NULL

Loop **B**: {f,g}

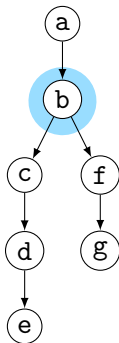
header: f; parent: NULL

Finding Natural Loops: Example

Control Flow Graph



Dominator Tree

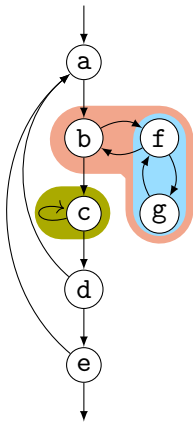


Loop Info

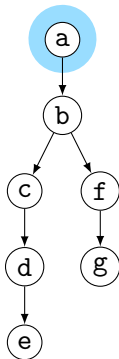
Loop **A**: {c}
header: c; parent: NULL
Loop **B**: {f,g}
header: f; parent: C
Loop **C**: {b,f,g}
header: b; parent: NULL

Finding Natural Loops: Example

Control Flow Graph



Dominator Tree

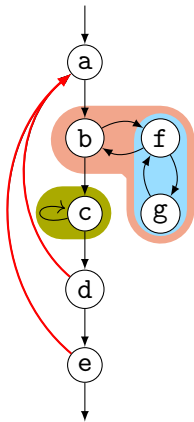


Loop Info

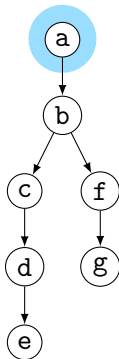
Loop **A**: {c}
header: c; parent: NULL
Loop **B**: {f,g}
header: f; parent: C
Loop **C**: {b,f,g}
header: b; parent: NULL

Finding Natural Loops: Example

Control Flow Graph



Dominator Tree

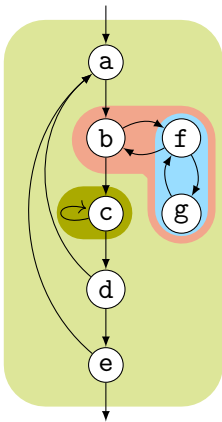


Loop Info

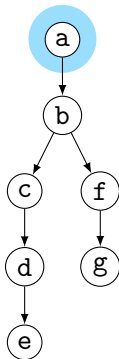
Loop **A**: $\{c\}$
header: `c`; parent: NULL
Loop **B**: $\{f, g\}$
header: `f`; parent: C
Loop **C**: $\{b, f, g\}$
header: `b`; parent: NULL

Finding Natural Loops: Example

Control Flow Graph



Dominator Tree



Loop Info

Loop **A**: {c}

header: c; parent: D

Loop **B**: {f,g}

header: f; parent: C

Loop **C**: {b,f,g}

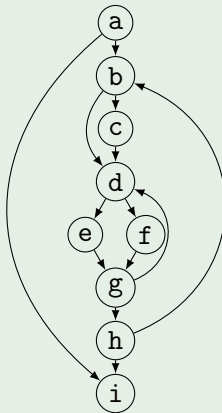
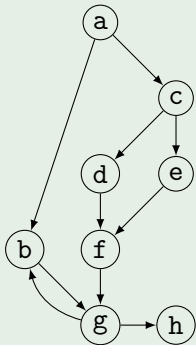
header: b; parent: D

Loop **D**: {a,b,c,d,e,f,g}

header: a; parent: NULL

Loop Analysis – Example

Apply the previous algorithm to find loops in the following CFGs (entry at a):



Loop Invariant Code Motion (LICM)

- ▶ Analyze loops, iterate over loop tree in post-order
 - ▶ I.e., visit inner loops first

Loop Invariant Code Motion (LICM)

- ▶ Analyze loops, iterate over loop tree in post-order
 - ▶ I.e., visit inner loops first
- ↑ Hoist:³⁰ iterate over blocks of loop in reverse post-order
 - ▶ For each movable inst., check for loop-defined operands
 - ▶ If not, move to preheader (create one, if not existent)
 - ▶ Otherwise, add inst. to set of values defined inside loop

³⁰<https://github.com/bytecodealliance/wasmtime/blob/bd6fe11/craneflight/codegen/src/licm.rs>

Loop Invariant Code Motion (LICM)

- ▶ Analyze loops, iterate over loop tree in post-order
 - ▶ I.e., visit inner loops first
- ↑ Hoist:³⁰ iterate over blocks of loop in reverse post-order
 - ▶ For each movable inst., check for loop-defined operands
 - ▶ If not, move to preheader (create one, if not existent)
 - ▶ Otherwise, add inst. to set of values defined inside loop
- ↓ Sink: Iterate over blocks of loop in post-order
 - ▶ For each movable inst., check for users inside loop
 - ▶ If none, move to unique exit (if existent)

³⁰<https://github.com/bytecodealliance/wasmtime/blob/bd6fe11/craneflight/codegen/src/licm.rs>

Loop Unswitch

- ▶ Hoist loop-invariant conditionals outside loop

```
while (...)  
  if (loopInvariantCond)  
    A  
  else  
    B  
  C
```

→

```
if (loopInvariantCond)  
  while (...)  
    A  
    C  
else  
  while (...)  
    B  
    C
```

Loop Unswitch

- ▶ Hoist loop-invariant conditionals outside loop

```
while (...)
  if (loopInvariantCond)
    A
  else
    B
  C
```

→

```
if (loopInvariantCond)
  while (...)
    A
  C
else
  while (...)
    B
  C
```

- ▶ Might duplicate parts of the loop $\rightsquigarrow$ needs heuristics
- ▶ Might lead to exponential code size increase

Loop Rotation

- ▶ Rotate loop condition to bottom
 - ▶ Essentially, convert while{} to if{do{}while}

```
define void @src() {  
    ; ...  
loopheader:  
    %iv = phi ...  
    %cond = ...  
    br i1 %cond, %body, %exit  
body:  
    ; ...  
    br %loopheader  
exit: ; ...  
}
```

→

```
define void @tgt() {  
    ; ...  
    %cond1 = ...  
    br i1 %cond1, %body, %exit  
body:  
    %iv = phi ...  
    ; ...  
    %cond2 = ...  
    br i1 %cond2, %body, %exit  
exit: ; ...  
}
```

Loop Strength Reduction

- ▶ Loop induction variables (e.g., loop counters) are often multiplied
 - ▶ E.g., for array indexing
- ▶ Strength reduction: replace “strong” multiply with “weak” addition
- ▶ Needs: analysis of loop induction variables and their recurrence
- ▶ Rewrite through replaced/extra loop induction variable
 - ▶ E.g., replace scaled index operation with pointer addition
- ▶ Information about ISA addressing modes beneficial

Analyses and Transformations – Summary

- ▶ Program Transformation critical for performance improvement
- ▶ Code not necessarily better
- ▶ Analyses are important to drive transformations
 - ▶ Dominator tree, loop detection, value liveness
- ▶ Important optimizations
 - ▶ Dead code elimination, sparse conditional constant propagation, common sub-expression elimination, loop-invariant code motion, loop unswitching, loop rotation, loop strength reduction

Analyses and Transformations – Questions

- ▶ Why is “optimization” a misleading name for a transformation?
- ▶ How to find unused code sections in a function's CFG?
- ▶ Why is a liveness-based DCE better than a simple, user-based DCE?
- ▶ Why is SCCP more powerful than separate DCE/constant prop.?
- ▶ What is a dominator tree useful for?
- ▶ What is the difference between an irreducible and a natural loop?
- ▶ How to find natural loops in a CFG?
- ▶ How does the algorithm handle irreducible loops?
- ▶ Why is sinking a loop-invariant inst. harder than hoisting?